FLAME PROPAGATION PROCESSES

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If a fuel mixture, first heated to the temperature T_0, is fed at a fixed velocity from the end of a semibounded cylindrical pipe there may be two stationary combustion processes. The first will occur when the rate of supply of fuel is low, and in this process the transfer of heat by conduction from the burnt particles to the fresh mixture will be most important. In the second process, which occurs when the fuel is supplied at high velocity, combustion is due to autoignition of the preheated fuel.

Papers [1, 2] were devoted to a study of these processes of stationary combustion. It was noted in [1] that the second process occurs rather frequently in modern technical equipment (jet engines).

The principal results in [2] were obtained by numerical integration of the corresponding boundary value problem using an electronic computer.

In the above-mentioned studies the combustion process is considered to be one-dimensional. It is well known, however, that both in processes of normal combustion and in processes with autoignition allowance for heat loss through the walls of the pipe is of great importance. For this reason it is desirable to study the process with the radial temperature distribution taken into account.

We have attempted to investigate the above-mentioned processes with additional consideration of the influence of the walls. First, this makes it possible to supplement our picture of the process by allowing for the possibility of quenching, which must be significant in an investigation of the stability of the combustion process. Second, the investigation makes use of linearization of the nonlinear function F(T); this makes it possible to obtain in simple form a criterion for different processes and to compute the distance which the fuel mixture must travel in the case of autoignition.

The combustion process in a semibounded cylindrical pipe of radius R is described (in the nonstationary regime) by the equation

$$\frac{\partial T}{\partial t} = a^2\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial x^2}\right) - u_0\frac{\partial T}{\partial x} + \delta F(T) \quad (1)$$

with the conditions

$$T|_{t=0} = 0, \quad T|_{x=0} = T_0, \quad T|_{r=R} = 0. \quad (2)$$

Here t is time, T temperature, x the coordinate along the pipe, r the radial coordinate, u_0 the rate of supply of fuel, and F(T) a function satisfying the conditions

$$F(T) = 0 \quad \text{for} \quad 0 \leqslant T \leqslant T^* \ (0 \leqslant T^* < T_1),$$
$$F(T) > 0 \quad \text{for} \quad T^* < T < T_1 \ F(T) = 0 \ \text{for} \ T \geqslant T_1.$$

The stationary regime is described by the equation

$$a^2\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial x^2}\right) - u_0\frac{\partial T}{\partial x} + \delta F(T) = 0,$$
$$T|_{x=0} = T_0, \qquad T_{r=R} = 0.$$

Simple considerations show that the influence of the walls is important. In fact we will consider the case of large u_0 and, as in [1, 2], we will neglect heat conduction in the direction of the x axis; then

$$u_0\frac{\partial T}{\partial x} = a^2\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right) + \delta F(T),$$
$$T|_{x=0} = T_0, \quad T|_{r=R} = 0.$$

This problem is similar (with replacement of t by x) to the nonstationary problem of thermal autoignition in an infinite cylinder of radius R with initial temperature T_0 and wall temperature equal to zero. It is known that there is a value $\delta = \delta^*$ such that for $\delta < \delta^*$ autoignition does not occur. However, in the one-dimensional theory for small δ combustion occurs in the autoignition regime.

Problem (3) with the nonlinear function F(T) is extremely difficult to study. It is possible to investigate this problem by numerical integration for specific forms of F(T) and different values of the parameters. However, the presence of radial terms complicates the numerical integration problem. In addition, the precise form of the function F(T) is usually unknown and precise values of many of the parameters are also unknown. It is therefore desirable to simplify the problem in such a way that it remains possible to obtain correct qualitative results and approximate quantitative estimates. It turns out that linearization of the function F(T) is perfectly admissible.

The choice of linearization method is extremely important.

The curve can be replaced either by a tangent at the point $(T_0, F(T_0))$ or by a straight line passing through the origin and the point $(T_0, F(T_0))$ or by a straight line passing through the origin in such a way that the area of the triangle on the segment $[0, T_1]$ is equal to the area under the F(T) curve. The choice of linearization method is determined by the specific form of the function F(T) and the value of T_0.

We replace F(T) by the function $\beta_1 T$. Then problem (3) assumes the form

$$a^2\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial x^2}\right) - u_0\frac{\partial T}{\partial x} + \beta T = 0,$$
$$T|_{x=0} = T_0, \qquad T|_{r=R} = 0, \qquad \beta = \delta\beta_1. \quad (4)$$

Problem (4) is solved by separation of variables. Assuming $T = \theta(r)\,X(x)$, we have

$$\theta'' = \frac{1}{r}\theta' + \lambda^2\theta = 0, \qquad \theta|_{r=R} = 0. \quad (5)$$

$$X'' = \frac{u_0}{a^2}X' + \left(\frac{\beta}{a^2} - \lambda^2\right)X = 0. \quad (6)$$

From (5) we have $\theta_k = J_0(\lambda_k r)$, where $J_0(z)$ is a zero-order Bessel function and λ_k are the roots of the equation $J_0(\lambda R) = 0$. The solution of (6) has the form

$$X_k(x) = \exp\left\{\left(\frac{u_0}{2a^2} - \left[\frac{u_0^2}{4a^4} + \left(\lambda_k^2 - \frac{\beta}{a^2}\right)\right]^{1/2}\right)x\right\}.$$

Thus, we obtain

$$T = T_0\sum_k A_k J_0(\lambda_k r)\,X_k(x). \quad (7)$$

The coefficients A_k are found from the equation

$$\sum_k A_k J_0(\lambda_k r) = 1.$$

Obviously, in (7) the first term is the most important. Therefore it may be assumed that

$$T = A_1 T_0 J_0(\lambda_1 r)\,e^{\nu x} \quad (8)$$

$$\left(\lambda_1 = \frac{b}{R},\ \nu = \frac{u_0}{2a^2} - \left[\frac{u_0^2}{4a^4} + \left(\lambda_1^2 - \frac{\beta}{a^2}\right)\right]^{1/2}\right).$$

Here b is the first root of the equation $J_0(z) = 0$. Considering Eq. (8), we obtain three cases:

1) If

$$\beta < \lambda_1^2 a^2 \quad (R < ab / \sqrt{\beta})$$

then as $x \to \infty$ the temperature $T \to 0$, that is, in this case damping occurs.

2) If

$$\beta > \lambda_1^2 a^2, \qquad \frac{u_0^2}{4a^4} + \left(\lambda_1^2 - \frac{\beta}{a^2}\right) \geqslant 0,$$

$$\text{or} \quad u_0 \geqslant 2a^2 \left(\frac{\beta}{a^2} - \lambda_1^2\right)^{1/2}$$

then as $x \to \infty$ the temperature $T \to \infty$, and the temperature remains finite at each point x. This case should be regarded as combustion in the autoignition regime. The unbounded increase of temperature as $x \to \infty$ is a result of the linear approximation. In actuality, the temperature cannot rise above the value T_1. We will find the distance from the end of the pipe to the point x with temperature T_1 (along the axis of the pipe). We have

$$T_1 = T_0 A_1 e^{\nu l}, \quad \text{or} \quad l = \frac{1}{\nu} \ln \frac{T_1}{T_0 A_1}. \tag{9}$$

At large u_0 the computations can be simplified, since in this case

$$\nu = \frac{u_0}{2a^2}\left\{1 - \left[1 + \left(\lambda_1^2 - \frac{\beta}{a^2}\right)\frac{4a^4}{u_0^2}\right]^{1/2}\right\} \approx$$

$$\approx \frac{u_0}{2a^2}\left[1 - \left(1 - \frac{\lambda_1^2 a^4 - \beta a^2}{u_0^2}\right)\right] = \frac{\beta - \lambda_1^2 a^2}{u_0}.$$

Then

$$l = \frac{u_0}{\beta - (a^2 b^2 / R^2)} \ln \frac{T_1}{T_0 A_1}. \tag{10}$$

3) If

$$\beta > \lambda_1 a^2, \qquad u_0 < 2a^2 \sqrt{(\beta / a^2) - \lambda_1^2} \tag{11}$$

there will be no stationary solution making physical sense. It is easy to establish from a study of the nonstationary linearized problem (1) that in this case even for x as close to zero as desired (that is, as close to the end of the pipe as desired) the temperature increases greatly with increase of t. This means that for u_0 satisfying (11) combustion will occur in the thermal conduction regime.

We introduce the dimensionless parameter

$$\mu = \frac{4a^2(\beta - \lambda_1^2 a^2)}{u_0^2}. \tag{12}$$

If $\mu \leq 0$, the process is damped. If $0 < \mu \leq 1$, combustion proceeds in the autoignition regime. If $\mu > 1$, combustion proceeds in the thermal conduction regime.

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